Minimal Spanning Trees

Lecture 32
Sections 7.1 - 7.3

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Outline

Networks and Trees

Definition (Network)

A network is a connected graph.

Networks and Trees

Definition (Network)

A network is a connected graph.

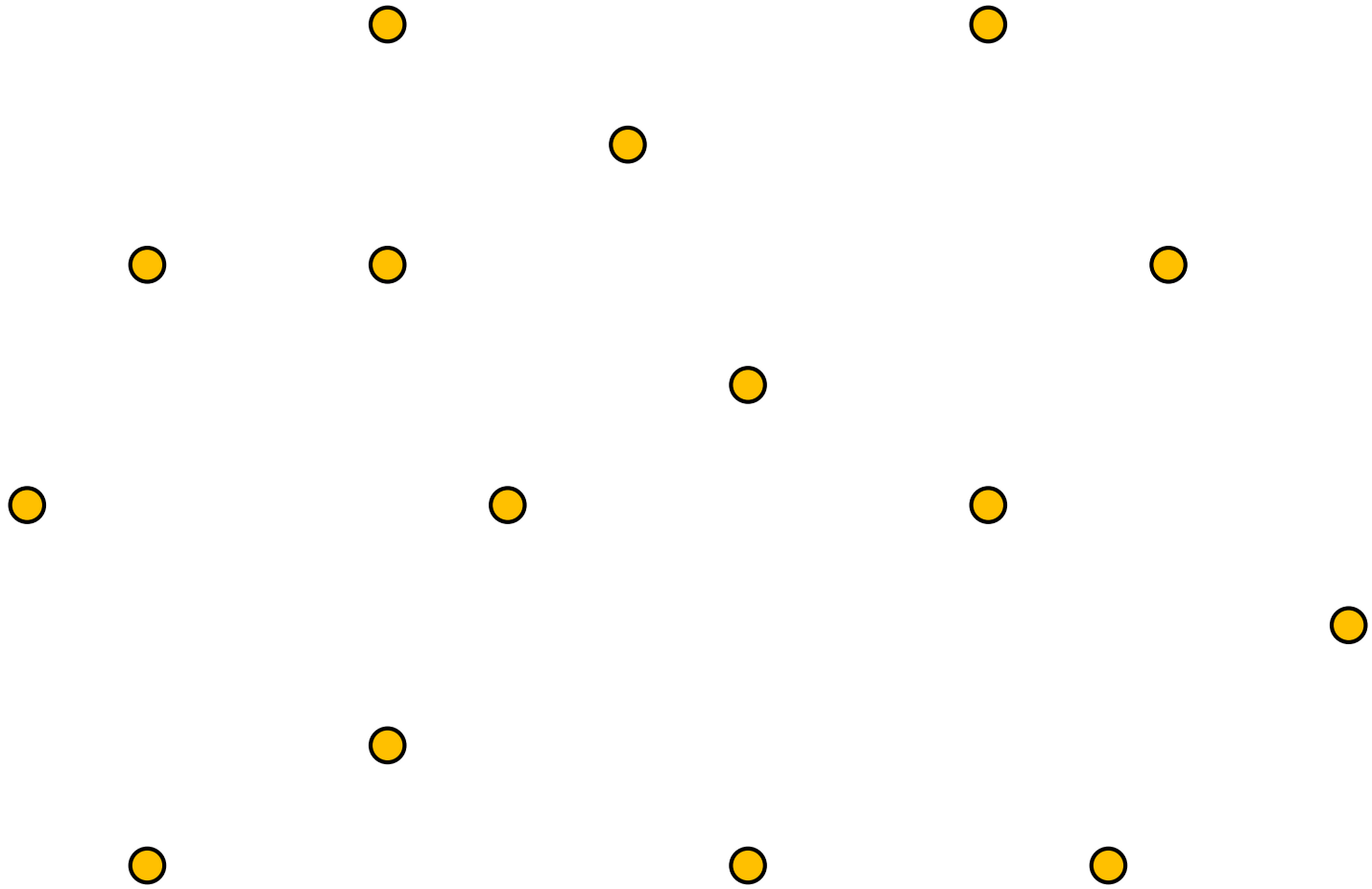
Definition (Tree)

A tree is a network that contains no circuits.

Properties of Trees

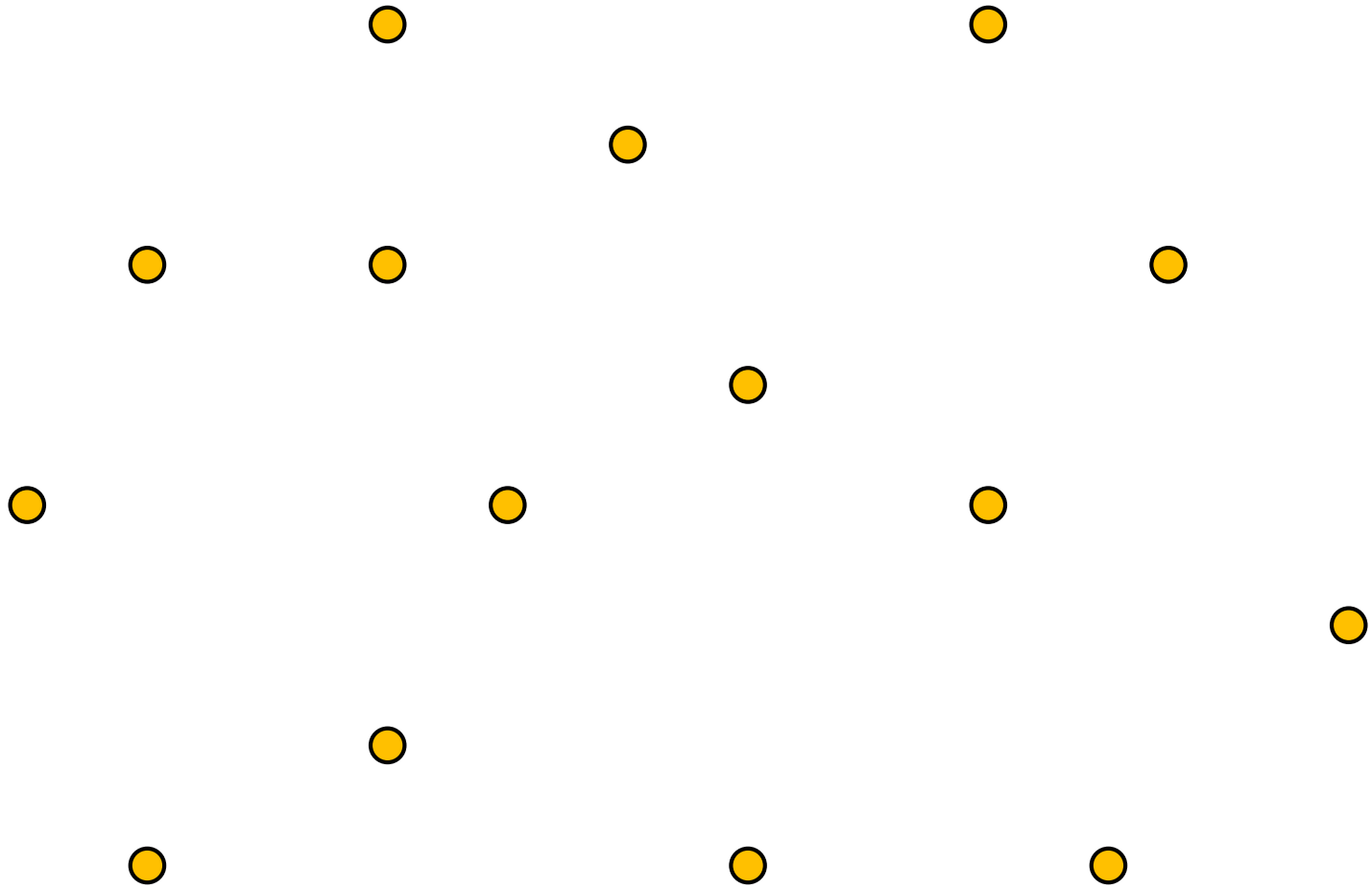
- If T is a tree with N vertices, then
  - T has exactly $N - 1$ edges.
  - If any edge of T is removed, then the resulting graph is not connected.

Properties of Trees



Add edges to connect the graph without making a circuit.

Properties of Trees



Erase edges to remove circuits without disconnecting the graph.

Properties of Trees

Theorem

A graph G with N vertices is a tree if any two of the following three properties hold.

(1) *T has exactly* $N - 1$ *edges.*

(2) *T is connected.*

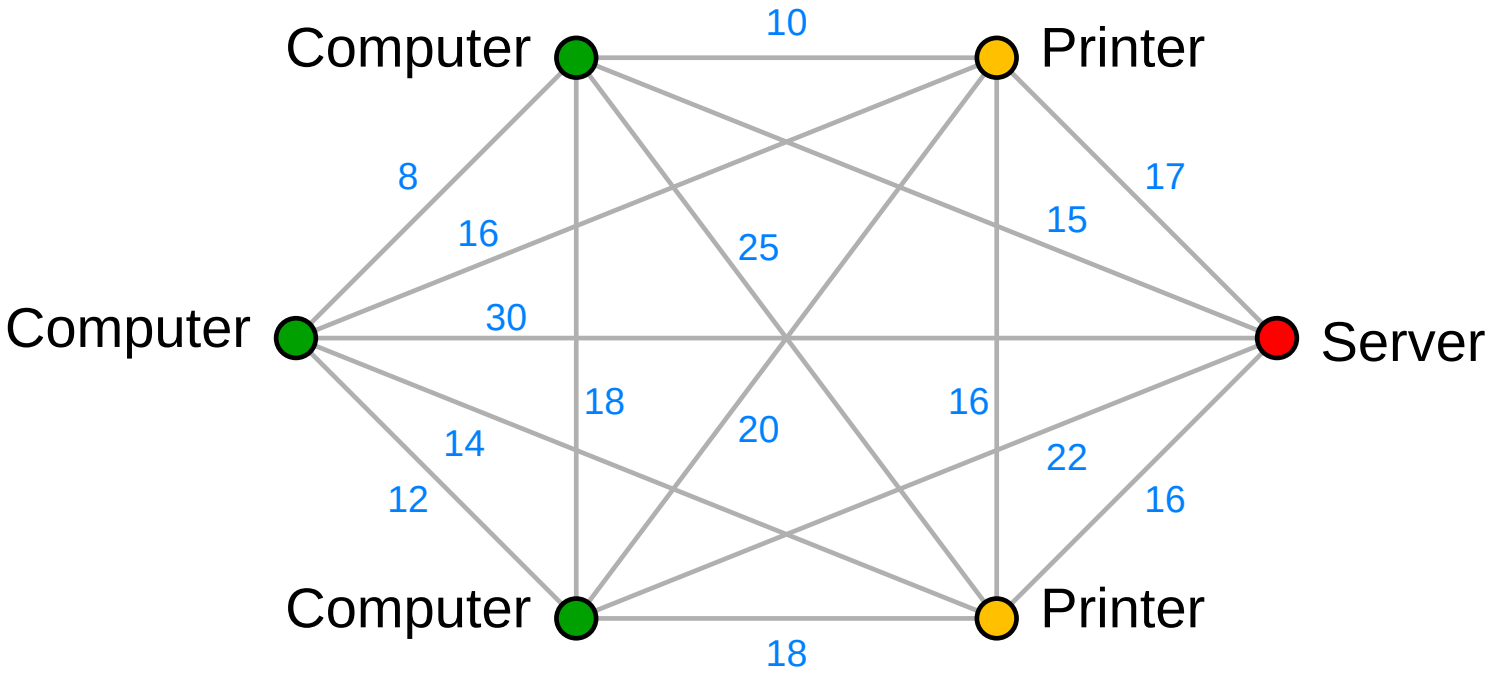
(3) *T contains no circuits.*

Outline

Examples

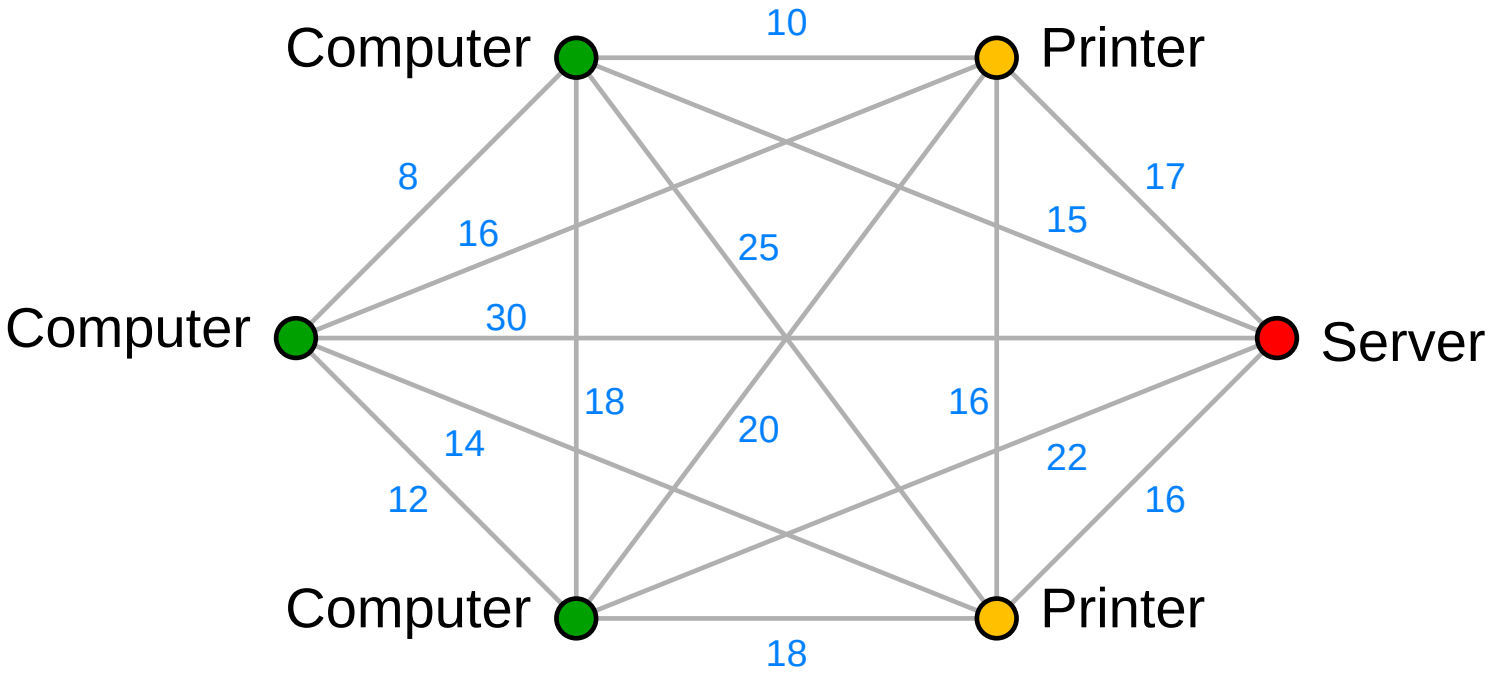
Suppose we want to connect devices in a network.

Examples



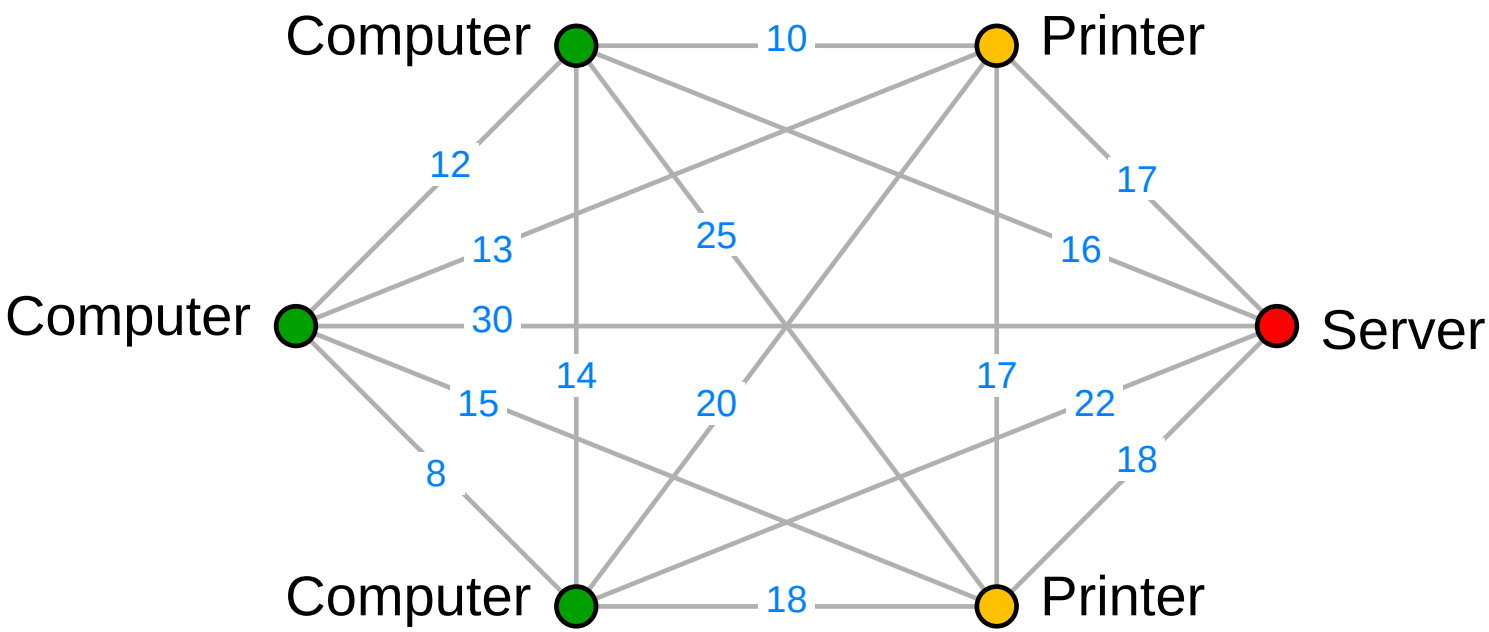
We want all the devices connected to each other.

Examples



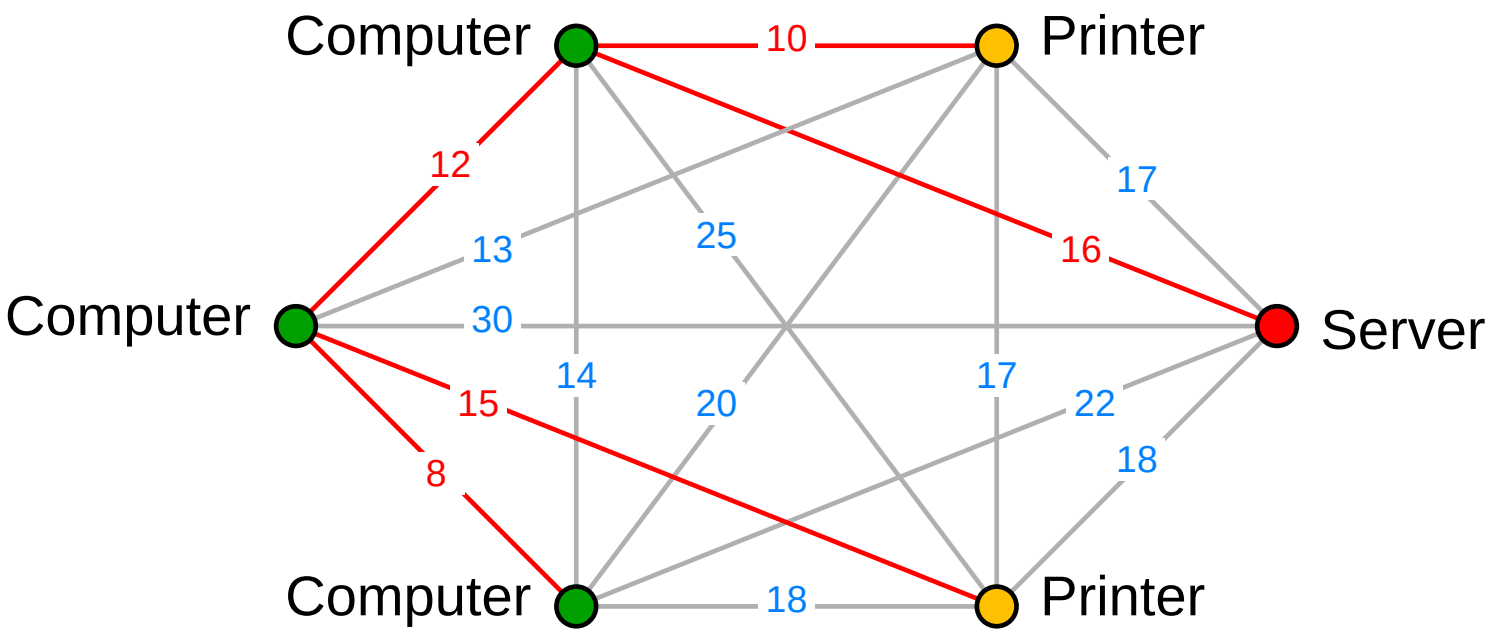
But no two devices need to be connected by more than a single path.

Examples



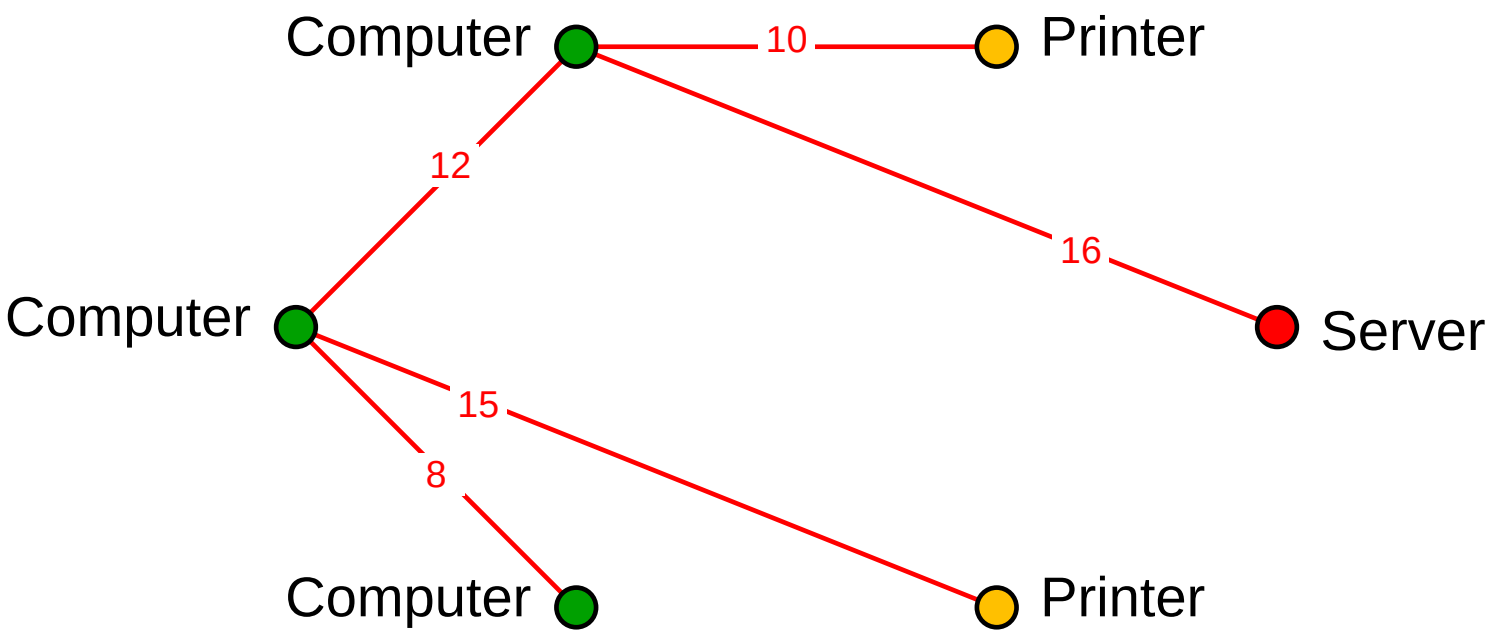
Given the distances, which lines do we keep?

Examples



The best choice is the tree of minimal total length

Examples



The total length of this tree is 61

Minimal Spanning Trees

Definition (Spanning Tree)

Given a graph G, a spanning tree of G is a subgraph T that is a tree and includes all the vertices of G.

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Definition (Minimal Spanning Tree)

Given a weighted graph G, a minimal spanning tree of G is a spanning tree T that has the smallest total weight of all possible spanning trees of G.

Outline

Kruskal's Algorithm

Kruskal's Algorithm

Given a graph G of N vertices, we may construct a minimal spanning tree as follows.

(1) Include all the vertices of G in T.

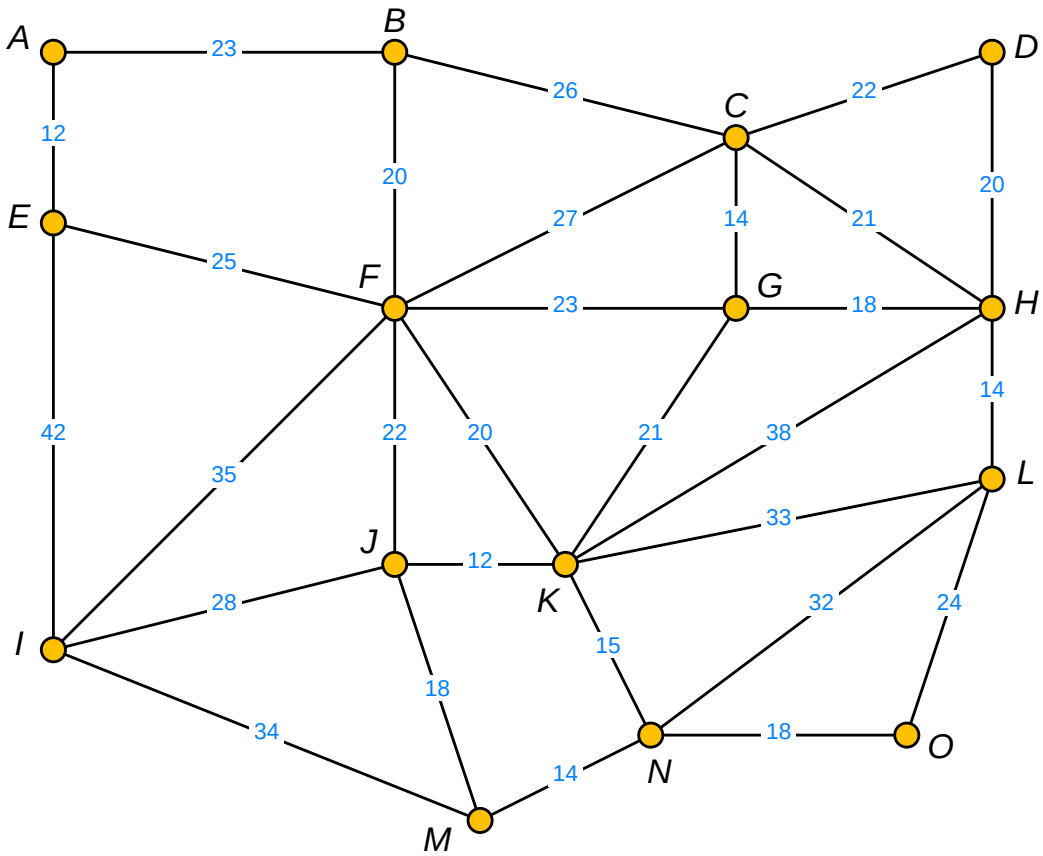
(2) Add to T the edge of minimal weight.

(3) Repeatedly add to T edges of minimal weight from among the remaining edges while being careful not to create a circuit.

(4) Once we have added $N - 1$ edges, T is a minimal spanning tree of G.

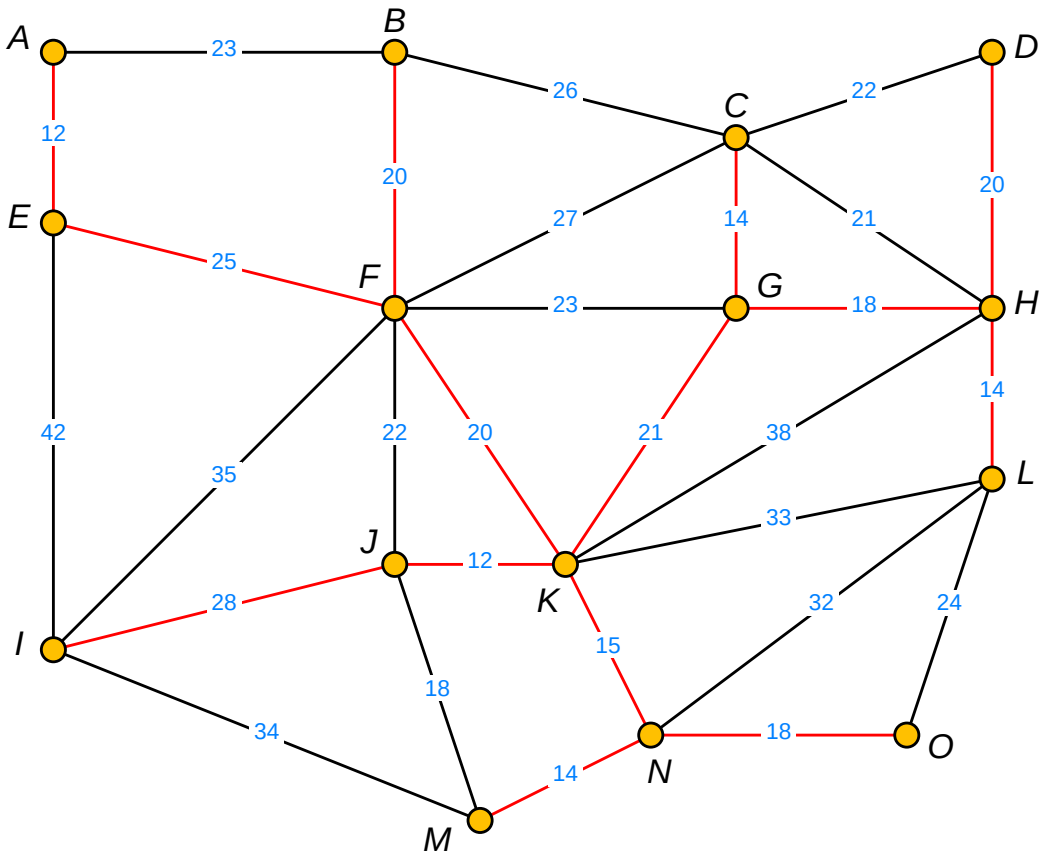
Outline

Power Grid



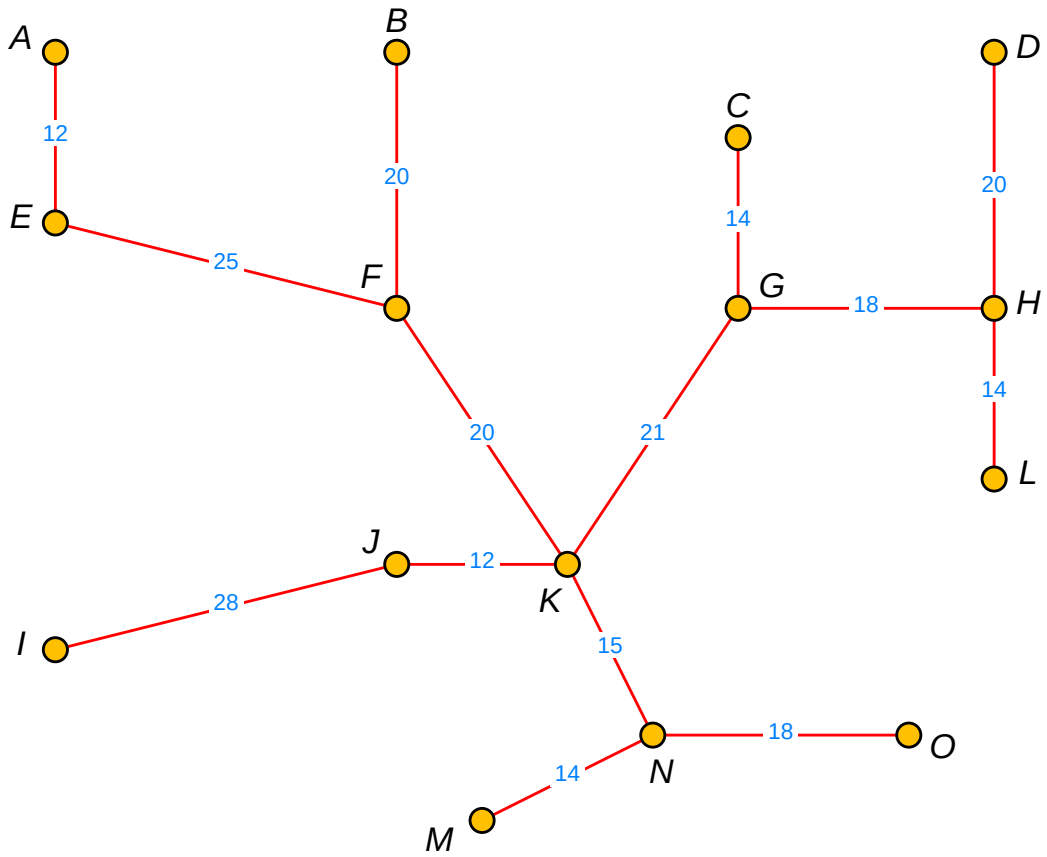
Given the possible links, find the minimal spanning tree.

Power Grid



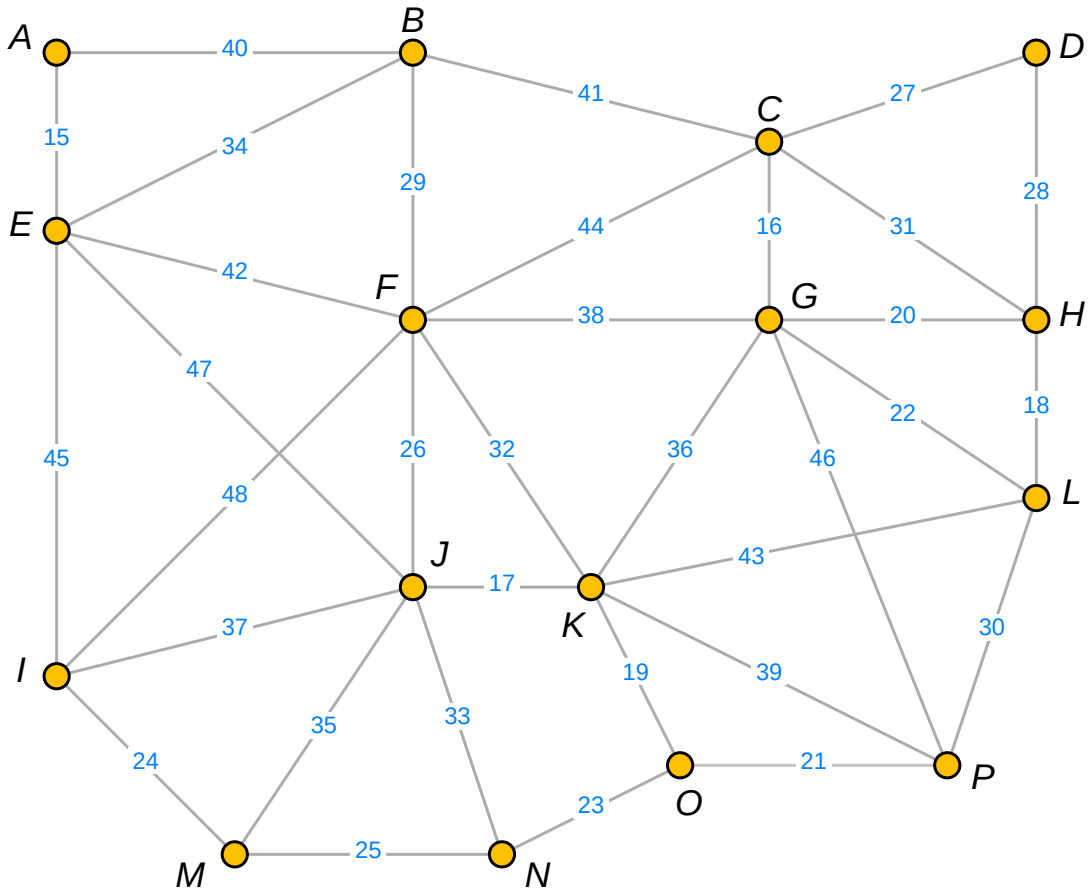
The minimal spanning tree.

Power Grid



Total weight is 251.

Practice



Find the minimal spanning tree.

Outline

Assignment

Assignment

- Chapter 7 Exercises 1, 3, 7, 35, 37, 39, 40, 45.